LOYOLA COLLEGE (AUTONOMOUS), CHENNAI- 6000034

M.Sc. DEGREE EXAMINATION- MATHEMATICS

I SEMESTER – NOVEMBER 2014

MT 1817 - ORDINARY DIFFERENTIAL EQUATIONS

TIME: Forenoon / Afternoon DATE: MAX: 100 MARKS

Answer all questions. Each question carries 20 marks.

1. (a) Prove that $x(t) = x_p(t) + x_h(t)$ is the general solution of $L[x(t)] = d(t)$ where $x_p(t)$ is any particular solution of $L[x(t)] = d(t)$ and $x_h(t)$ is the general solution of the homogeneous equation $L[x(t)] = 0$. **(5)**

(OR)

(b) Prove that $uL(v) - vL(u) = a_0(t)\frac{d}{dt}W[u,v] + a_1(t)W[u,v]$, where *u*, *v* are twice differentiable functions and a_0, a_1 are continuous on *I*. **(5)**

(c) Explain the method of variation of parameters. **(15)**

(OR)

(d) Determine whether the given sets of functions are linearly dependent or independent.
(i) e^x , e^{-x} (ii) $1+x,\ 1-x, 1-3x$ (iii) $\sin x,\ \sin 2x, \sin 3x$ on $I = [0,2\pi]$
(iv) $1+x,\ x^2+x, 2x^2-x-3$ and (v) $1,\ x,\ x^2, \dots, x^n$ **(15)**

2. (a) Find the Legendre polynomial of first four orders. **(5)**

(OR)

(b) State and prove Rodrigure's formula. **(5)**

(c) Show that $\frac{1}{\sqrt{1-2tx+t^2}} = \sum_{l=0}^{\infty} t^l P_l(x)$ if $|t| < 1$ and $|x| \le 1$. **(15)**

(OR)

(d) Solve by Frobenius method: $x^2\frac{d^2y}{dx^2} + (x^3 - 2x)\frac{dy}{dx} + 2y = 0$. **(15)**

3. (a) When n is an integer (positive or negative), show that $J_{-n}(x) = (-1)^n J_n(x)$. **(5)**

(OR)

(b) State and prove any two recurrence relations on Bessel's function. **(5)**

(c) State and prove the integral representation of Bessels's function. **(15)**

(OR)

(d) With usual notation, prove that $P_n(x) = {}_2F_1(-n, n+1; 1; \frac{1-x}{2})$. **(15)**

4. (a) Using the method of successive approximations, solve the initial value problem $x' = -x, x(0) = 1,\ t \geq 0.$ **(5)**

(OR)

(b) For the parameters, λ and μ $(\lambda \neq \mu)$, let *x* and *y* be the corresponding solutions of the Strum-Liouville problem such that $[pW(x,y)]_A^B = 0$. Prove that $\int_A^B r(s)x(s)y(s)ds = 0.$ **(5)**

(c) State Green's function and prove that $x(t)$ is a solution of $L[x(t)] + f(t) = 0$, $a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds.$ **(15)**

(OR)

(d) State and prove Picard's theorem for boundary value problem. **(15)**

5. (a) Prove that the null solution of equation $x' = A(t)x$ is stable if and only if a positive constant *k* exists such that $|\phi(t)| \leq k, t \geq t_0.$ **(5)**

(OR)

(b) Explain asymptotically stable solution by an example. **(5)**

(c) Explain the stability of $x' = A\,x$ by Lyapunov's method. **(15)**

(OR)

(d) State and prove the fundamental theorems on the stability of non-autonomous systems. **(15)**

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